

Project Management and Budget Development

Projects can be large or small. The larger they become the greater the possibility that something could be forgotten up front that causes a delay or additional cost. Project management concepts can be applied to any size project to reduce the risk of delays and additional costs.

Project management techniques can also help one develop a budget or verify that the available funds are sufficient for the project.

Definition:

Project management is the discipline of organizing and managing resources in such a way that these resources deliver all the work required to complete a project within defined scope, quality, time and cost constraints.

Resources. When there are conversations of resources, often the only resource considered is money. However, in project management resources can include money, people, materials, energy, space and other non-traditional items.

A **project** is a temporary and one-time endeavor undertaken to create a unique product or service, that brings about beneficial change or added value.

Traditional Approach to Defining Stages

Projects have stages. The following configuration (of five stages) is the traditional way of looking at the project stages.

1. project initiation stage
2. project planning or design stage
3. project execution or production stage
4. project monitoring and controlling systems
5. project completion stage

Not all projects will visit every stage. Some projects can be terminated before they reach completion. And for some projects it is not applicable to have a monitoring and controlling stage. On the other hand, some projects will go through steps 2 and 3 multiple times. For this class, a subset of these stages will be examined – those which can most readily be applied to real life situations.

What to do

Step 1 – Define your project. Although this step sounds simple, it is not usually simple. What may be obvious to you may not be obvious to the person who is assigned the task.

For example, a project manager may indicate that carpet should be used on the first floor of the new house. The project manager may think that it is obvious that carpet should not

be used in the kitchen, but unless he/she clearly states this, the person responsible for floors may put carpet in the kitchen.

Projects need to be performed and delivered under certain constraints. Traditionally, these constraints have been listed as **scope, time, and cost**. This is also referred to as the *project management triangle*, where each side represents a constraint. One side of the triangle cannot be changed without impacting the others.

The **scope** constraint refers to what must be done to produce the project's end result. The **time** constraint refers to the amount of time available to complete a project. The **cost** constraint refers to the budgeted amount available for the project. These three constraints are often competing constraints: increased scope typically means increased time and increased cost, and tight time constraint could mean increased costs and reduced scope, and a tight budget could mean increased time and reduced scope.

The discipline of project management is about providing the tools and techniques that enable the project manager to organize their work to meet these initial constraints.

Thus, step one is to describe the initial scope, time, and cost constraints of the project. If you are not sure of the time and cost constraints. You can begin with the scope (assume the other two are unconstrained). In the planning stage, when you reach the point of listing each task, you can estimate time and costs for each task. As you add up the time and costs, you will begin to determine if they are manageable or too high.

Step 2 – A Work Breakdown Structure (WBS) is an exhaustive, hierarchical (from general to specific) tree structure of deliverables and tasks that need to be performed to complete a project. The purpose of the WBS is to identify terminal elements (the actual items to be done in a project). Therefore, the WBS serves as the basis for much of project planning.

Features of a WBS

Size – the size of the WBS should generally not exceed 100-200 terminal elements (if more terminal elements seem to be required, use subprojects).

The 100% Rule – states that the WBS includes 100% of the work defined by the project scope and captures all deliverables.

Mutually exclusive elements – there is no overlap in scope definition between two elements of a WBS.

Level of detail (granularity) – continue dividing work into smaller elements until a time and cost can be assigned to a terminal element

Class Exercise:

Develop a hierarchical (from general to specific) WBS tree structure for painting a room

Step 3 – Using the WBS, estimate the time and cost for each terminal element. If you do not have good estimates for each element, research is appropriate. Also determine the dependencies between the elements. Which ones must be accomplished before others?

Step 4 – Using the information from step 3 (a list of all activities required, the duration of each activity, and the dependencies between activities), it is time to develop a schedule or model for the project. Often flowcharts (see page 771 of the text) or Gantt charts (see page 8-4 of the notes) are used.

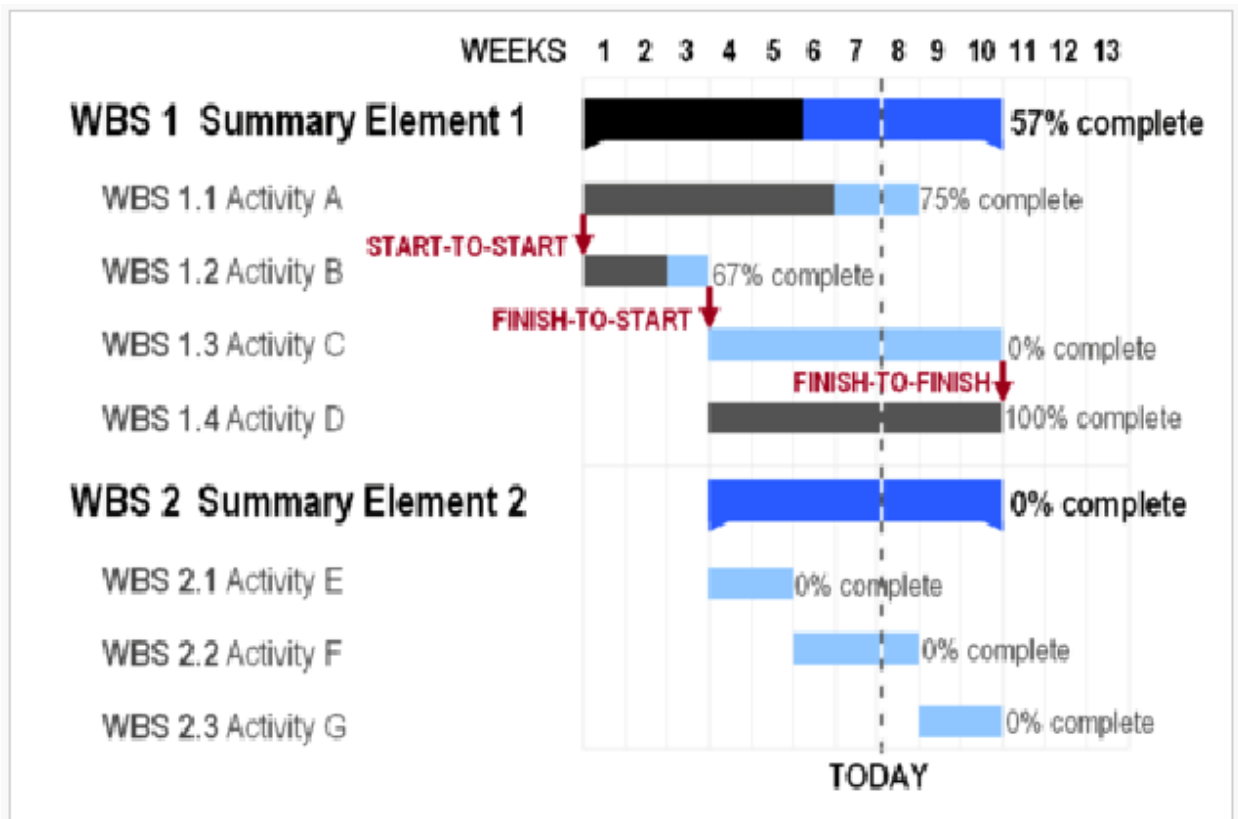
Class Exercise:

Develop a flowchart using the activities listed in the WBS for painting a room

The **critical path** is the sequence of project network activities with the longest overall duration, determining the shortest time possible to complete the project. Any delay of an activity on the critical path directly impacts the planned project completion date. A project can have several, parallel near critical paths. An additional parallel path through the network with the total durations shorter than the critical path is called a sub-critical or non-critical path.

These results allow managers to prioritize activities for the effective management of project completion.

A Gantt chart is a popular type of bar chart that illustrates a project schedule. Gantt charts illustrate the start and finish dates of the terminal elements of a project. Recall terminal elements were determined from the work breakdown structure. Some Gantt charts also show the dependency relationships between activities.



Gantt chart showing three kinds of schedule dependencies (in red) and percent complete indications.

Step 5 – Developing the Budget. Thus far, we have defined the scope of the project and have handled the time constraints by developing a schedule (flow chart or Gantt chart). The final step is to deal with the budget and the timing of the need of funds. Each step thus far, is required to complete the financial aspect. The WBS is needed to determine the cost and required budget of the project. The schedule is needed to determine the timing of the needed fund (i.e. the funds may all be required up front or more likely some funds are required up front and others are needed throughout the project life).

Upon completion of the WBS, an estimated cost is made for each terminal element. As suggested, some research may be required to estimate the costs. These cost estimates can be summarized in table form to become the **overall budget**. The costs are not usually listed in chronological order but are usually listed in categories of like items. For example, material would be listed together even though the materials may be needed at different times in the project or are ordered at different times. Page 8-5 contains a sample overall budget.

Sample Capital Campaign Budget

Build-out of Small Theater in an Existing Building

EXPENSES

Phase I

Architectural Fees	\$25,000.00
Consultants/Inspections	\$2,000.00
Engineering Fees	\$8,500.00
Building Permit	\$2,200.00
Contingency @ 10%	\$3,770.00
	\$41,470.00

Phase II

Demolition	\$1,000.00
Steel Reinforcement	\$4,500.00
Concrete work	\$5,000.00
Steel Grid Installation	\$6,000.00
Carpentry	\$29,000.00
Insulation and Sealants	\$1,000.00
Fire doors and frames	\$4,000.00
Other doors/windows	\$3,100.00
Fire Protection	\$5,500.00
Fire Extinguishers	\$500.00
Finish Hardware	\$4,100.00
Drywall over wood studs	\$20,000.00
Plumbing	\$23,264.00
Bathroom Fixtures	\$4,144.00
Heating/Ventilation	\$16,000.00
Electrical	\$32,124.00
Ceramic tile	\$2,750.00
Floor covering & base	\$3,200.00
Painting	\$3,200.00
Contractors Fee	\$12,000.00
Contingency @ 15%	\$27,057.00
	\$207,439.00

Furniture & Fixtures

Lighting Equipment	\$47,500.00
Lighting Grid	\$22,000.00
Security/Alarm System	\$2,000.00
Seating	\$4,100.00
Sound System	\$6,000.00
Contingency @ 10%	\$8,160.00
	\$89,760.00

Related/Office Costs

Insurance	\$3,650.00
Fundraising Costs	\$5,500.00
Organizational Costs	\$11,000.00
Contingency @ 10%	\$2,015.00
	\$22,165.00

TOTAL COSTS	\$360,834.00
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The overall budget is usually developed without regard to time. A second chart, a **monthly budget**, is helpful to demonstrate when funds are needed. This chart usually has a column for each month of the project. If funds are needed in one area during a specific month, the amount is listed in the appropriate month column. When developing the schedule, take note of when the funds are needed in addition to the initiation of the work.

Description	Jan	Feb	Mar	Apr	May	Jun	Jul
Variable Costs							
<i>material</i>							
sticks	6,465					10,375	
rawhides			4,601				
beads		944					
wristbands			1,590			1,590	
tracts					2,884		
trifolds	1,026						
follow-up books						6,000	
follow-up cards					86		
meat processing							
banners, etc						1,244	
<i>travel</i>							
gas	200	200	50	50	50	250	200
food						240	
lodging						400	
air fare							
<i>promotions</i>							
breakfast letter	44	44	44	44	44	44	44
newsletter	1,367			1,367			
fund raiser letter			500			500	
magazine							
luncheon			9,063				
picnic							3,039
<i>short term missions</i>							
shipping	60	60	60	60	60	60	60
<i>unknowns (10%)</i>	1,052	997	2,555	1,425	1,412	1,997	1,395
Total Variable Costs	10,213	2,245	18,463	2,946	4,536	22,700	4,738

Class Exercise:

Develop a budget for painting a room.

Solving Financial Problems

Everyday we are faced with financial questions. Should we keep the old car and pay the repair bill or should we trade the old car in and buy a new car? Should we take out a student loan and start school this semester or should we try to save money and start next year. Should we take out a loan to pay off the credit card or should we try to pay an extra \$100 per month until the balance is paid off? This unit will help you use math to deal with the financial problems which you face in everyday life.

There are many financial instruments in business. This lecture will address many of the basic concepts and financial instruments such as budgeting, simple interest, compounded interest, points, car loans, mortgages, savings accounts, CDs, and credit cards.

A Personal Monthly Budget

The idea of budgeting is to ensure that the incoming funds are sufficient to cover the outgoing funds. Making a budget means keeping track of how much money is coming in and how much money is going out and then deciding what adjustments need to be made. The most common form of a personal budget is the monthly budget. However, there can be yearly, quarterly, or weekly budgets. Additionally there can be personal, family or business budgets. The concepts are similar to a personal monthly budget with adjustments to accommodate for the entity or time.

In this section, a personal monthly budget formation will be considered.

Five Step Budget

1. List all your monthly income.
2. List all your monthly expenses. You may want to include a general category for unexpected expenses because there are always unpredictable events in life: like being invited to a wedding and needed to purchase a gift or taking a sick child to the doctor. In the class exercises, the income and bills are given, but for real life, you many have to keep track of where the funds are going by listing what you spend money on over a month or several months.
3. Consider income or expenses that do not come monthly but rather come quarterly or once a year. Prorate these amounts to be considered in your monthly budget. For example, expenses that do happen monthly are tuition, books, vacation and holiday gifts.
4. Subtract the total expenses from the total income to determine if the budget is in good shape (i.e. there is more incoming funds than outgoing funds) or if the budget needs adjustments (i.e. there is more outgoing funds than incoming funds).
5. Make adjustments to the budget as needed. Adjustments can be in either of two forms: increasing the income or decreasing the expenses. It is usually difficult to

increase the income so most often the focus of adjusting the budget is to determine which activities will have to cease in order to decrease the amount of funds going out.

Prorate the Amounts which are not Monthly

Definition:

Prorate means to divide, distribute, or assess proportionately. Thus, to prorate income (or an expense) which is not given monthly to a monthly budget, means to divide the income (or expense) proportionately and distribute the proportions to the appropriate month.

Example 1: This might best be explained with an example. Once a year, you receive \$1,200 from an investment which your parents set up for you when you were a baby. Your instructor tells you to create a monthly budget and to prorate this once a year income. What do you do? The income of \$1,200 must be proportionately distributed over the 12 months.

$\$1,200 / 12 \text{ months} = \100 per month.

Thus, if you spread this money out to have an equal amount to use each month, you would use \$100 each month.

Class Exercise: You are taking classes at WBC. You want to take two classes in the fall semester for which the college will probably charge you \$2,400 and you want to take one class in the spring for which the college will probably charge you \$1,296. How would you prorate these costs for a monthly budget?

Develop a Budget

We are going to use the five step budget process to develop a budget.

Example 2: Virginia never seems to have enough money at the end of the month. So she decides to create a monthly budget so that she can understand what the problem is.

Step 1: List all monthly income:

Salary from her regular job: \$2,080
Salary from keeping her niece every Saturday: \$120

Step 2: List all monthly expenses:

Rent:	\$700	Health Ins:	\$140
Cable:	\$ 50	Auto Ins:	\$ 75
Internet:	\$ 35	Groceries:	\$300
Cell Phone:	\$ 90	Leisure:	\$250
Utilities:	\$100	Offerings:	\$ 50
Car Pmt:	\$300	Unexpected:	\$ 50
Gas for Car:	\$120		

Step 3: Prorate none monthly income and expenses. Virginia does not have any other income but she does spend money at Christmas time and takes a class each semester at WBC.

Christmas Spending: \$480
Two classes at WBC: \$2400
WBC Books: \$180

Step 4: Subtract the total expenses from the total income.

Step 5: Make adjustment as needed.

Simple Interest versus Compounding Interest

Definition:

The **principal** in financial formulas is the balance upon which interest is paid.

Interest is the price paid for the use of credit or money. **Interest rate** is the rate paid expressed as a percentage.

The **prime interest rate** (also called prime rate) is an interest rate formally announced by a bank to be the lowest available at a particular time to its most credit-worthy customers.

Simple interest is interest paid only on the original principal, and not on any interest added at later dates.

Compound interest is interest paid both on the original principal and on all interest that has been added to the original principal.

Calculating Simple Interest

Example 2: A deposit of \$1,000 is made to the bank. The bank promises to pay simple interest in one year of 5% on the deposit. What is the amount of the interest if 5% of \$1,000 is paid in interest?

$$\begin{aligned}5\% \text{ of } \$1,000 &= \text{what} \\0.05 \times \$1,000 &= \text{interest} \\\$50 &= \text{interest}\end{aligned}$$

Thus, after one year the bank would give the depositor \$50. Because it is simple interest the bank would give the \$50 of interest to the person and not add it to the account

If the depositor left the money in the account for a second year the bank would pay simple interest of 5% on the \$1,000. After the second year the bank would give the depositor \$50 dollars in interest. Because it is simple interest, again, the bank would give the \$50 of interest to the person and not add it to the account.

Thus, in two years a total of \$100 is received in interest.

Calculating Compound Interest

Although it is important to consider simple interest in order to understand other financial concepts, almost no-one is using simple interest but instead uses compound interest. With compounding, interest is paid on both the principal and on all interest that has been added to the principal. This is good when you are receiving the interest and not very good when you are paying the interest. Let us consider the same example only instead of receiving simple interest for two years, let us receive compound interest for two years.

Example 3: A deposit of \$1,000 is made to the bank. The bank promises to pay compound interest (compounded yearly) of 5% on the deposit. How much interest was paid after two year?

Year 1

$$\begin{aligned}5\% \text{ of } \$1,000 &= \text{what} \\0.05 \times \$1,000 &= \text{interest} \\\$50 &= \text{interest}\end{aligned}$$

After year one the interest of \$50 is added to the principal. Thus during the second year the bank is paying 5% interest on \$1,050 (which comes from \$1,000 + \$50).

Year 2

$$\begin{aligned}5\% \text{ of } \$1,050 &= \text{what} \\0.05 \times \$1,050 &= \text{interest} \\\$52.50 &= \text{interest}\end{aligned}$$

After the second year, notice that the bank paid \$52.50 instead of \$50.00. Thus, in two years a total of \$102.50 is received in interest. The difference comes about because interest was paid on the interest as well as on the original principal. Notice that the two situations were not identical. In the first case the depositor had \$50 to spend after the first year and in the second case the \$50 had to remain in the bank to collect interest.

It is important to understand compounding in order to calculate projected savings and loan payments.

Calculating Compound Interest Using a Formula and Excel

Using a Formula

Compound Interest Formula for Interest Paid Once a Year

$$A = P \times (1 + \text{APR})^Y$$

A = accumulated balance after Y years
 P = starting principal
 APR = annual percentage rate (as a decimal)
 Y = number of years

Example 4: A deposit of \$1,000 is made to the bank. The bank promises to pay compound interest (compounded yearly) of 5% on the deposit. The depositor leaves all the money in the bank for 10 years. What will be the accumulated balance after 10 years and how much interest is paid in the 10 years?

P = \$1,000
 APR = 0.05
 Y = 10 years

$$A = P \times (1 + \text{APR})^Y$$

$$A = 1,000 \times (1 + 0.05)^{10}$$

$$A = 1,000 \times (1.05)^{10}$$

$$A = 1,000 \times (1.05)^{10}$$

$$A = 1,000 \times 1.6288946$$

$$A = \$1,628.8946$$

The accumulated balance after 10 years will be \$1,628.89.
 The interest paid is \$628.89 (which comes from \$1,628.89 - \$1,000).

Using Excel

The following shows the use of a spreadsheet in Excel to solve the same problem. Notice the black boxes, this row represents the initial deposit and no time has passed and no interest has been paid. The row with “end year 1” shows that \$50.00 interest is accumulated at the end of year one and the account balance is \$1,050.00. Thus at the end of year ten, the account balance is \$1,628.89. The remaining problem is to calculate total interest paid. The summation button is used to sum the column of interest paid to come to a total of \$628.89.

	A	B	C	D
1	Year	Interest Paid	Balance	Interest Rate
2			1,000.00	5%
3	end year 1	50.00	1,050.00	5%
4	end year 2	52.50	1,102.50	5%
5	end year 3	55.13	1,157.63	5%
6	end year 4	57.88	1,215.51	5%
7	end year 5	60.78	1,276.28	5%
8	end year 6	63.81	1,340.10	5%
9	end year 7	67.00	1,407.10	5%
10	end year 8	70.36	1,477.46	5%
11	end year 9	73.87	1,551.33	5%
12	end year 10	77.57	1,628.89	5%
13	TOTAL:	628.89		

The following table contains the formulas used to calculate the above table.

	A	B	C	D
1	Year	Interest Paid	Balance	Interest Rate
2			1000	0.05
3	end year 1	=C2*D2	=C2+B3	=D2
4	end year 2	=C3*D3	=C3+B4	=D3
5	end year 3	=C4*D4	=C4+B5	=D4
6	end year 4	=C5*D5	=C5+B6	=D5
7	end year 5	=C6*D6	=C6+B7	=D6
8	end year 6	=C7*D7	=C7+B8	=D7
9	end year 7	=C8*D8	=C8+B9	=D8
10	end year 8	=C9*D9	=C9+B10	=D9
11	end year 9	=C10*D10	=C10+B11	=D10
12	end year 10	=C11*D11	=C11+B12	=D11
13	TOTAL:	=SUM(B3:B12)		

There are advantages to using Excel over the hand formula. For example, you can look at each year’s balance and decide how many years you want the investment to remain. Perhaps you only needed to save \$1,500. Thus, you can withdrawal the funds after 9 years. You can also change the percentage rate by changing one cell and the entire table adjusts for your one change.

Compound Interest Paid More than Once a Year

What happens if the interest is not compounded yearly but instead it is compounded quarterly, or monthly, or weekly or daily? In fact, compounding quarterly, monthly and daily is common.

Using a Formula

Compound Interest Formula for Interest Paid n Times per Year

$$A = P \left(1 + \frac{APR}{n} \right)^{nY}$$

- A = accumulated balance after Y years
- P = starting principal
- APR = annual percentage rate (as a decimal)
- n = number of compounding periods per year
- Y = number of years

Example 5: Let us use the same original deposit of \$1,000. But now the bank promises to pay quarterly compounding at an annual percentage rate of 5%. The depositor leaves all the money in the bank for 10 years. What will be the accumulated balance after 10 years and how much interest is paid in the 10 years?

- P = \$1,000
- APR = 0.05
- n = 4
- Y = 10 years

$$A = P \left(1 + \frac{APR}{n} \right)^{nY}$$

$$A = 1,000 \left(1 + \frac{0.05}{4} \right)^{4 \cdot 10}$$

$$A = 1,000(1 + 0.0125)^{40}$$

$$A = 1,000(1.0125)^{40}$$

$$A = 1,000(1.643619)$$

$$A = 1,643.62$$

The accumulated balance after 10 years will be \$1,643.62. The interest paid is \$643.62 (which comes from \$1,643.62 - \$1,000). Notice by changing from compounding yearly to compounding quarterly there is an increase of \$14.73. What do you think would happen if the compounding were changed to monthly or daily?

Using Excel

The following shows the use of a spreadsheet in Excel to solve the same problem. Notice the black boxes, this row represents the initial deposit and no time has passed and no interest has been paid. The row with “end Qtr 1” shows that \$12.50 interest is accumulated at the end of the first quarter and the account balance is \$1,012.50. Thus at the end of year ten, the account balance is \$1,643.62. The remaining problem is to calculate total interest paid. The summation button is used to sum the column of interest paid to come to a total of \$643.62.

Year	Time	Interest Paid	Balance	Interest Rate	APR/n
			1,000.00	5%	0.0125
	end Qtr 1	12.50	1,012.50	5%	0.0125
	end Qtr 2	12.66	1,025.16	5%	0.0125
	end Qtr 3	12.81	1,037.97	5%	0.0125
end year 1	end Qtr 4	12.97	1,050.95	5%	0.0125
	end Qtr 5	13.14	1,064.08	5%	0.0125
	end Qtr 6	13.30	1,077.38	5%	0.0125
	end Qtr 7	13.47	1,090.85	5%	0.0125
end year 2	end Qtr 8	13.64	1,104.49	5%	0.0125
	end Qtr 9	13.81	1,118.29	5%	0.0125
	end Qtr 10	13.98	1,132.27	5%	0.0125
	end Qtr 11	14.15	1,146.42	5%	0.0125
end year 3	end Qtr 12	14.33	1,160.75	5%	0.0125
	end Qtr 13	14.51	1,175.26	5%	0.0125
	end Qtr 14	14.69	1,189.95	5%	0.0125
	end Qtr 15	14.87	1,204.83	5%	0.0125
end year 4	end Qtr 16	15.06	1,219.89	5%	0.0125
	end Qtr 17	15.25	1,235.14	5%	0.0125
	end Qtr 18	15.44	1,250.58	5%	0.0125
	end Qtr 19	15.63	1,266.21	5%	0.0125
end year 5	end Qtr 20	15.83	1,282.04	5%	0.0125
	end Qtr 21	16.03	1,298.06	5%	0.0125
	end Qtr 22	16.23	1,314.29	5%	0.0125
	end Qtr 23	16.43	1,330.72	5%	0.0125
end year 6	end Qtr 24	16.63	1,347.35	5%	0.0125
	end Qtr 25	16.84	1,364.19	5%	0.0125
	end Qtr 26	17.05	1,381.25	5%	0.0125
	end Qtr 27	17.27	1,398.51	5%	0.0125
end year 7	end Qtr 28	17.48	1,415.99	5%	0.0125
	end Qtr 29	17.70	1,433.69	5%	0.0125
	end Qtr 30	17.92	1,451.61	5%	0.0125
	end Qtr 31	18.15	1,469.76	5%	0.0125
end year 8	end Qtr 32	18.37	1,488.13	5%	0.0125
	end Qtr 33	18.60	1,506.73	5%	0.0125
	end Qtr 34	18.83	1,525.57	5%	0.0125
	end Qtr 35	19.07	1,544.64	5%	0.0125
end year 9	end Qtr 36	19.31	1,563.94	5%	0.0125
	end Qtr 37	19.55	1,583.49	5%	0.0125
	end Qtr 38	19.79	1,603.29	5%	0.0125
	end Qtr 39	20.04	1,623.33	5%	0.0125
end year 10	end Qtr 40	20.29	1,643.62	5%	0.0125
Total Interest Paid =		643.62			

The following table contains the formulas used to calculate the above table.

	A	B	C	D	E	F
1	Year	Time	Interest Paid	Balance	Interest Rate	APR/n
2				1000	0.05	=E2/4
3		end Qtr 1	=D2*F2	=D2+C3	=E2	=F2
4		end Qtr 2	=D3*F3	=D3+C4	=E3	=F3
5		end Qtr 3	=D4*F4	=D4+C5	=E4	=F4
6	end year 1	end Qtr 4	=D5*F5	=D5+C6	=E5	=F5
7		end Qtr 5	=D6*F6	=D6+C7	=E6	=F6
8		end Qtr 6	=D7*F7	=D7+C8	=E7	=F7
9		end Qtr 7	=D8*F8	=D8+C9	=E8	=F8
10	end year 2	end Qtr 8	=D9*F9	=D9+C10	=E9	=F9
11		end Qtr 9	=D10*F10	=D10+C11	=E10	=F10
12		end Qtr 10	=D11*F11	=D11+C12	=E11	=F11
13		end Qtr 11	=D12*F12	=D12+C13	=E12	=F12
14	end year 3	end Qtr 12	=D13*F13	=D13+C14	=E13	=F13
15		end Qtr 13	=D14*F14	=D14+C15	=E14	=F14
16		end Qtr 14	=D15*F15	=D15+C16	=E15	=F15
17		end Qtr 15	=D16*F16	=D16+C17	=E16	=F16
18	end year 4	end Qtr 16	=D17*F17	=D17+C18	=E17	=F17
19		end Qtr 17	=D18*F18	=D18+C19	=E18	=F18
20		end Qtr 18	=D19*F19	=D19+C20	=E19	=F19
21		end Qtr 19	=D20*F20	=D20+C21	=E20	=F20
22	end year 5	end Qtr 20	=D21*F21	=D21+C22	=E21	=F21
23		end Qtr 21	=D22*F22	=D22+C23	=E22	=F22
24		end Qtr 22	=D23*F23	=D23+C24	=E23	=F23
25		end Qtr 23	=D24*F24	=D24+C25	=E24	=F24
26	end year 6	end Qtr 24	=D25*F25	=D25+C26	=E25	=F25
27		end Qtr 25	=D26*F26	=D26+C27	=E26	=F26
28		end Qtr 26	=D27*F27	=D27+C28	=E27	=F27
29		end Qtr 27	=D28*F28	=D28+C29	=E28	=F28
30	end year 7	end Qtr 28	=D29*F29	=D29+C30	=E29	=F29
31		end Qtr 29	=D30*F30	=D30+C31	=E30	=F30
32		end Qtr 30	=D31*F31	=D31+C32	=E31	=F31
33		end Qtr 31	=D32*F32	=D32+C33	=E32	=F32

Planning Ahead with Compound Interest

Example 6: Suppose you could make a single deposit in an investment with an interest rate of APR = 5%, compounded annually, and leave it there for the next 18 years. How much would you have to deposit now to realize \$100,000 after 18 years?

$$\begin{aligned} \text{APR} &= 0.05 \\ Y &= 18 \\ A &= \$100,000 \\ P &= \text{principal} \quad \leftarrow \text{This is what we want to find.} \end{aligned}$$

$$A = P \times (1 + \text{APR})^Y$$

Solve for P

$$P = \frac{A}{(1 + \text{APR})^Y}$$

$$P = \frac{\$100,000}{(1 + 0.05)^{18}} = \frac{\$100,000}{(1.05)^{18}} = \$41,552.07$$

Depositing \$41,552.07 now will yield the desired \$100,000 in 18 years – assuming that the 5% APR doesn't change and that you make no withdrawals or additional deposits.

Example 7: Repeat this problem but with an interest rate of APR = 7% and monthly compounding. Compare the results.

This time we must solve for P in the compound interest formula for more than one year.

$$A = P \left(1 + \frac{\text{APR}}{n} \right)^{nY}$$

Solve for P

$$P = \frac{A}{\left(1 + \frac{\text{APR}}{n} \right)^{nY}}$$

$$P = \frac{A}{\left(1 + \frac{\text{APR}}{n} \right)^{nY}} = \frac{\$100,000}{\left(1 + \frac{0.07}{12} \right)^{12 \times 18}} = \frac{\$100,000}{(1.005833333)^{216}} = \$28,469.43$$

With a 7% APR and monthly compounding, you can reach \$100,000 in 18 years by depositing about \$28,469.43 today. This is over \$13,000 less than the above example.

Solving Financial Problems (Continued)

Thus far we have discussed preparing a monthly budget and compound interest. We looked at investing a sum of money and then calculating the compound interest over a period of time. If we wanted to save money for retirement, or for a child's college, you could deposit a lump of money today and let it grow through the power of compound interest. But what if you don't have a large lump to start with? For most people, a more realistic way to save is to deposit smaller amounts of money on a regular basis. For example, you might put \$100 a month into savings. This next section will look at calculating the results of repeated savings with compound interest.

Savings Plan

Let us look at an example to demonstrate a savings plan. You deposit \$100 into a savings account at the end of each month. Your account pays interest monthly at an annual rate of APR = 12%.

- You begin with \$0 in the account. At the end of month 1, you make the first deposit of \$100.
- At the end of month 2, you receive the monthly interest on the \$100 which has been in the account for one month. In addition, you make your monthly deposit of \$100. What is your balance?

$$\begin{aligned} & \text{Prior balance} + \text{interest} + \text{new deposit} \\ &= \$100 + \$100x\frac{0.12}{12} + \$100 \\ &= \$100 + \$100x0.01 + \$100 \\ &= \$100 + \$1.00 + \$100 \\ &= \$201 \end{aligned}$$

- At the end of month 3, you receive the monthly interest on the \$201 which has been in the account for one month. In addition, you make your monthly deposit of \$100. What is your balance?

$$\begin{aligned} & \text{Prior balance} + \text{interest} + \text{new deposit} \\ &= \$201 + \$201x\frac{0.12}{12} + \$100 \\ &= \$201 + \$201x0.01 + \$100 \end{aligned}$$

$$= \$201 + \$2.01 + \$100$$

$$= \$303.01$$

This process continues each month as long as you consistently make a \$100 deposit on the last day of the month and the interest remains at 12% paid monthly.

Here is a table showing the monthly accumulation of funds.

End of ...	Prior Balance	Interest	End-of Month Deposit	New Balance
Month 1	\$0.00	\$0.00	\$100.00	\$100.00
Month 2	\$100.00	\$1.00	\$100.00	\$201.00
Month 3	\$201.00	\$2.01	\$100.00	\$303.01
Month 4	\$303.01	\$3.03	\$100.00	\$406.04
Month 5	\$406.04	\$4.06	\$100.00	\$510.10
Month 6	\$510.10	\$5.10	\$100.00	\$615.20

Calculating Accumulated Savings Using a Formula

There is a formula which mathematicians have developed to help solve financial problems of this sort. This formula has a couple of assumptions which we must be aware of.

1. The deposit is always made at the end of the month.
2. The deposit period and the compound period are the same. I.E. if deposits are made monthly, interest also is calculated and paid monthly.

$$A = PMT \times \frac{\left[\left(1 + \frac{APR}{n} \right)^{(nY)} - 1 \right]}{\left(\frac{APR}{n} \right)}$$

A = accumulated savings plan balance
 PMT = regular deposit (payment) amount
 APR = annual percentage rate (as a decimal)
 n = number of deposit (payment) periods per year
 Y = number of years

$$A = \$100 \times \frac{\left[\left(1 + \frac{0.12}{12} \right)^{(12 \times 0.5)} - 1 \right]}{\left(\frac{0.12}{12} \right)}$$

$$A = \$100x \frac{[(1.01)^6 - 1]}{0.01}$$

$$A = \$100x \frac{1.061520 - 1}{0.01}$$

$$A = \$100x \frac{0.061520}{0.01}$$

$A = \$100x6.1520 = \615.20 Notice the result is the same as the table above.

Calculating Accumulated Savings Using Excel

Just as before, it is often helpful to use Excel. In this section we will use two techniques in Excel to calculate accumulated savings. First we will develop a detailed spreadsheet which calculates the balance for each period. Second we will use an Excel formula to calculate accumulated savings.

Developing a Detailed Spreadsheet in Excel.

	A	B	C	D	E	F	G
		Prior Balance	Interest	End-of Month Deposit	New Balance		Interest per Month
1	End of ...						
2	Month 1	\$0.00	\$0.00	\$100.00	\$100.00		0.01
3	Month 2	\$100.00	\$1.00	\$100.00	\$201.00		
4	Month 3	\$201.00	\$2.01	\$100.00	\$303.01		
5	Month 4	\$303.01	\$3.03	\$100.00	\$406.04		
6	Month 5	\$406.04	\$4.06	\$100.00	\$510.10		
7	Month 6	\$510.10	\$5.10	\$100.00	\$615.20		
8							

	A	B	C	D	E	F	G
		Prior Balance	Interest	End-of Month Deposit	New Balance		Interest per Month
1	End of ...						
2	Month 1	0	0	100	=B2+C2+D2		=0.12/12
3	Month 2	=E2	=B3*G\$2	=D2	=B3+C3+D3		
4	Month 3	=E3	=B4*G\$2	=D3	=B4+C4+D4		
5	Month 4	=E4	=B5*G\$2	=D4	=B5+C5+D5		
6	Month 5	=E5	=B6*G\$2	=D5	=B6+C6+D6		
7	Month 6	=E6	=B7*G\$2	=D6	=B7+C7+D7		
8							

Using an Excel Formula to Calculate Accumulated Savings

First, Some Excel Background

Before jumping into using the Excel formula, a couple of basic Excel concepts must be addressed.

1. Excel represents cash flows *in* to a person's pocket as positive numbers and cash flows *out* of a person's pocket as negative numbers. For example, when you calculate the payments on a loan, the situation is as follows:

- The amount of the loan is entered as a positive value because this is the money you'll receive from the bank.
- The monthly payment is a negative value because this is money that you'll be paying out.

2. Most interest rates are expressed as an annual rate. However, Excel needs the interest rate per payment period in order to calculate properly. For example, if you're calculating for a loan with monthly payment, you need the monthly interest rate. As you know, you can easily get this number by dividing the annual interest rate by 12, the number of months in a year.

3. As you have learned, to solve financial equations, you need to provide variables such as interest rate, deposit amount (payment amount), compounding period, etc. Excel often requires an additional variable which we have not discussed yet. Excel calls this variable "type." For Excel the variable "type" tells the function whether deposits (payments) are applied at the end of the period or the beginning of the period. A value of 0 indicates the end of the period. A value of 1 indicates the beginning of the period. The default is 0 (i.e. if you do not specify a type, then Excel will assign a type of 0).

Recall the example we have been using thus far in today's class: "You deposit \$100 into a savings account at the end of each month. Your account pays interest monthly at an annual rate of APR = 12%." Notice the deposit of \$100 is made at the end of each month. For Excel you would set the type equal to 0 because the deposit is applied at the end of the period.

Second, Using the Future Value (FV) Formula to Calculate Accumulated Savings

Excel has several financial formulas. The one which we need for calculating accumulated savings is the Future Value or FV Formula. As with any formula there are variables, and you must tell Excel the values of the variables before it can make any calculations. For Future Value the variables are rate, nper, pmt, pv, and type. The syntax of this function is:

$FV(\text{rate}, \text{nper}, \text{pmt}, [\text{pv}], [\text{type}])$

RATE is the interest rate **per period**. Remember, interest rates are usually given as an annual rate. If the period is something other than one year, a calculation is required to obtain the RATE **per period**.

NPER is the total number of payment (or deposit) periods in the life of the plan. When the life of the plan is greater than one year, then the NPER is calculated by taking the

Term (the amount of time that interest is paid) and multiplying it by the Period (the point in time when interest is paid or earned). For example, if you were going to keep the savings plan for a three-year term with 12 monthly interest payments, the NPER would be 3×12 , or 36 payment (or deposit) periods.

PMT is the payment (or deposit in our case) made each period.

PV stands for Present Value. The Present Value is the amount in the account before the payments or deposits begin. In our case there was nothing in the account before the first deposit, so the $PV = 0$. However, if the account already had \$1,085 in it before you began your regimented savings plan of \$100 per month, then the PV would be equal to \$1,085. The default is 0 (i.e. if you do not specify a PV, then Excel will assume zero).

TYPE tells the function whether deposits (payments) are applied at the end of the period or the beginning of the period. A value of 0 indicates the end of the period. A value of 1 indicates the beginning of the period. The default is 0 (i.e. if you do not specify a type, then Excel will assign a type of 0).

Let us return to our problem. You deposit \$100 into a savings account at the end of each month. Your account pays interest monthly at an annual rate of $APR = 12\%$. What is the balance at the end of 6 months? To use Excel, we must determine the rate, nper, pmt, pv, and type.

$$\text{rate} = \text{annual interest rate} / 12 \text{ months} = 0.12/12 = 0.01$$

$$\text{nper} = 6 \text{ (6 deposits in 6 months)}$$

$$\text{pmt} = \$100$$

$$\text{pv} = 0 \text{ (there is nothing in the account when we began)}$$

$$\text{type} = 0 \text{ (the deposits are made at the end of each month)}$$

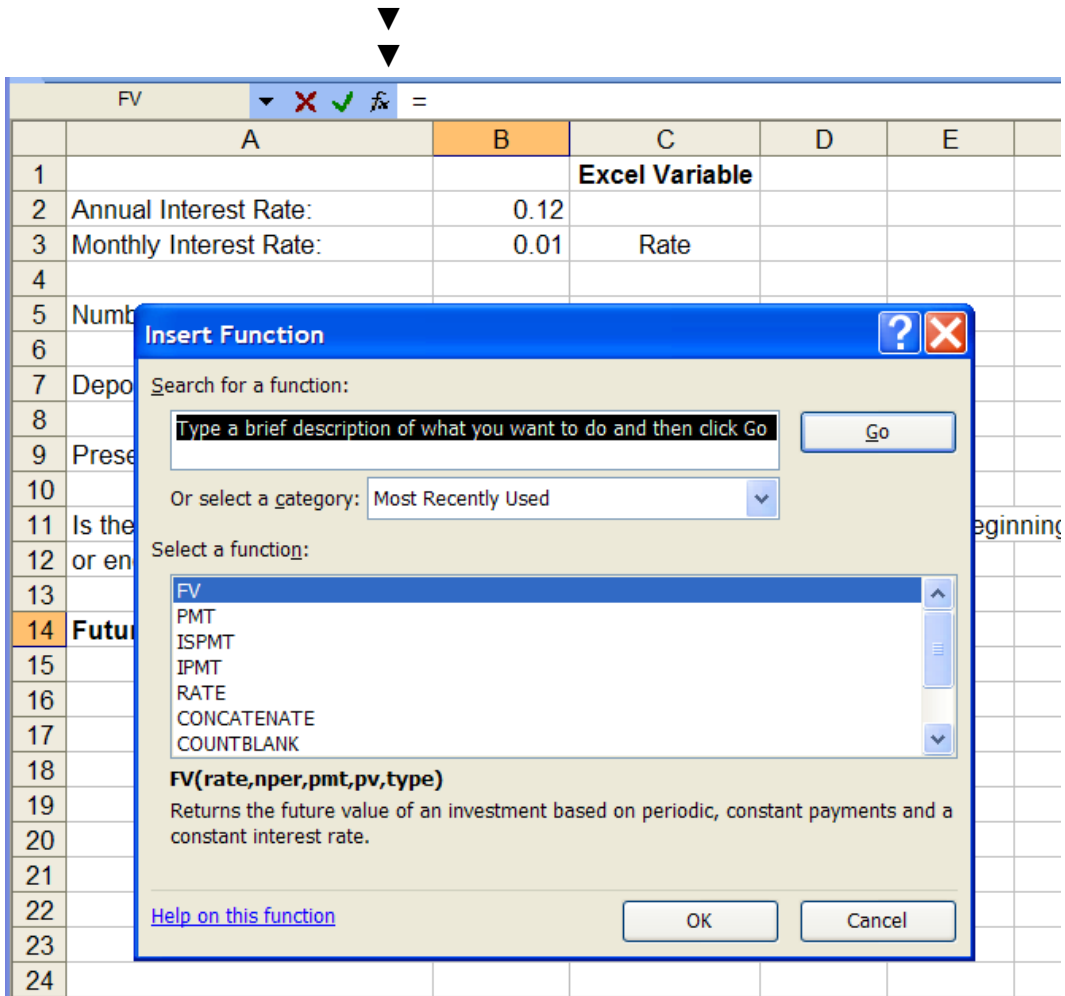
We are ready to use Excel to calculate the future value of our savings.

	A	B	C	D	E	F	G
1			Excel Variable				
2	Annual Interest Rate:	0.12					
3	Monthly Interest Rate:	0.01	Rate				
4							
5	Number of months (periods)	6	nper				
6							
7	Deposit	\$(100.00)	pmt				
8							
9	Present Value	0	pv				
10							
11	Is the deposit at the beginning	0	type (0 for end of month or 1 for beginning of month)				
12	or end of the month?						
13							
14	Future Value	\$615.20	FV				
15							

To keep things straight, type in each variable name and calculate or input the value of each variable. Although the annual interest is not used by the FV formula, I have included it for completeness and to use in calculating the rate ($= B2/12$). Input the value of each variable, remember, use formulas when ever possible.

Why is the deposit a negative number? We can tell it is negative because the parenthesis. Recall that Excel represents cash flows *in* as positive numbers and cash flows *out* as negative numbers. Although safely in the bank, Excel considers this deposit flowing out of your pocket and into the banks.

Once all the variables are entered on the spreadsheet, we are ready to calculate Future Value. First, click on the *fx* button shown here.



The Insert Function screen will appear. Select the FV option shown by the blue line. Then click the OK button.

The following new screen will appear. Click on the box following the word “Rate.” Either type in the cell number for the monthly interest rate (B3) or click on cell B3. Continue with this process until all the boxes have the correct address. Then click the OK button.

	A	B	C	D	E	F	G
1			Excel Variable				
2	Annual Interest Rate:	0.12					
3	Monthly Interest Rate:	0.01	Rate				
4							
5	Number of months (periods)	6	nper				
6							
7	Deposit	\$(100.00)	pmt				
8							
9	Present Value	0	pv				
10							
11	Is the deposit at the beginning	0	type (0 for end of month or 1 for beginning of month)				
12	or end of the month?						
13							
14	Future Value						
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							
26							
27							
28							
29							
30							
31							

Function Arguments

FV

Rate B3 = 0.01

Nper B5 = 6

Pmt B7 = -100

Pv B9 = 0

Type B11 = 0

= 615.201506

Returns the future value of an investment based on periodic, constant payments and a constant interest rate.

Type is a value representing the timing of payment: payment at the beginning of the period = 1; payment at the end of the period = 0 or omitted.

Formula result = \$615.20

[Help on this function](#) OK Cancel

As you can see the formula result is \$615.20, which is consistent with our earlier calculations.

General Comments about Savings Plans

The discussions thus far on savings plans develop the idea of having a bank account and making regular deposits to the bank account. However, there are other savings plans such as Individual Retirement Accounts (IRAs), 401k plans, and employee pension plans. All of these can be treated the same way to calculate an estimated accumulated savings. The term *estimated* is used because not all the variables are fixed. For example, the IRA and 401k plans are usually based on the stock market; and there is not a fixed interest rate. In this case a projected interest rate would be used. This could be based on the average rate of return for the last 5 years for a particular IRA or 401k plan.

Planning Ahead with Savings Plans

Most people start savings plans with a particular goal in mind, such as saving enough for retirement or enough to buy a new car in a few years. For planning ahead, the question may be: Given a financial goal, what regular payments are needed to reach the goal.

Now in this case we need a formula which when solved gives the payment (PMT) instead of accumulated savings (A). Using algebra and manipulating the accumulated savings formula, a PMT formula can be derived. The formula to calculate the payment (PMT) is:

$$PMT = \frac{A\left(\frac{APR}{n}\right)}{\left[\left(1 + \frac{APR}{n}\right)^{(nY)} - 1\right]}$$

Since this formula was derived from the accumulated savings formula, it has the same assumptions:

1. The deposit is always made at the end of the month.
2. The deposit period and the compound period are the same. I.E. if deposits are made monthly, interest also is calculated and paid monthly.

Example: You want to build a \$100,000 college fund in 18 years by making regular, end-of-month deposits. Assuming an APR of 7%, calculate how much you should deposit monthly. How much of the final value comes from actual deposits and how much from interest?

$$PMT = \frac{100,000\left(\frac{0.07}{12}\right)}{\left[\left(1 + \frac{0.07}{12}\right)^{(12 \cdot 18)} - 1\right]}$$

$$PMT = \frac{100,000(0.00583333)}{\left[(1.00583333)^{(216)} - 1\right]}$$

$$PMT = \frac{583.3333}{[3.512539 - 1]} = \frac{583.3333}{2.512539} = \$232.17$$

$$A = \$100,000$$

$$APR = 0.07$$

$$n = 12$$

$$Y = 18$$

- Note that the deposits are made at the end of the month, so our formula is acceptable.
- We will also need to find an investment location which compounds monthly.

Monthly payments of \$232.17 will give you \$100,000 after 18 years.

During the 18 years, how much did you deposit?

$$\frac{18 \text{ yr}}{1} \times \frac{12 \text{ mo}}{\text{yr}} \times \frac{\$232.17}{\text{mo}} = \$50,148.72$$

How much total interest was earned?

$$\$100,000 - \$50,148.72 = \$49,851.53$$

Just over half of the \$100,000 comes from your actual deposits; the rest is the result of compound interest.

Using Excel to Calculate the Payment

As you would expect, Excel has a formula to calculate the payment and it is called PMT. The syntax for PMT is:

$$\text{PMT}(\text{rate}, \text{nper}, \text{pv}, [\text{fv}], [\text{type}])$$

Let us look at our problem again. You want to build a \$100,000 college fund in 18 years by making regular, end-of-month deposits. Assuming an APR of 7%, calculate how much you should deposit monthly. How much of the final value comes from actual deposits and how much from interest?

rate = 0.07 / 12 (remember Excel needs a monthly rate and not the annual interest rate)

$$\text{nper} = \frac{18\text{yr}}{1} \times \frac{12\text{mo}}{\text{yr}} = 216 \text{ monthly payments}$$

pv = 0 (there is currently nothing invested)

fv = \$100,000

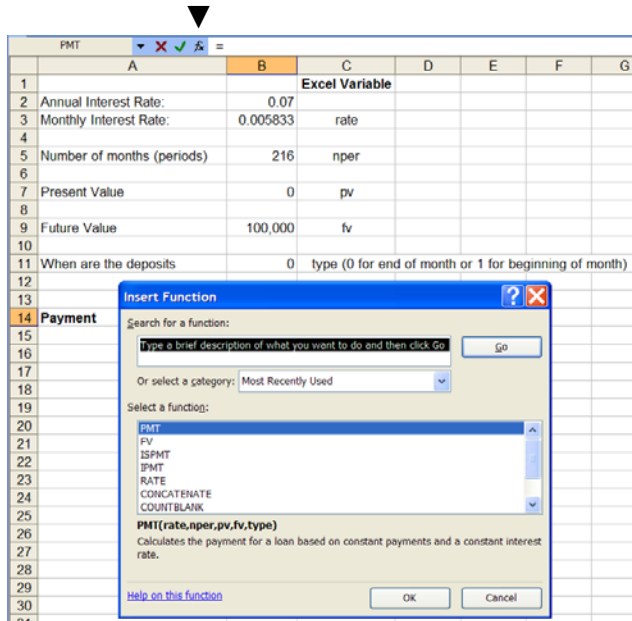
type = 0 (the deposits are made at the end of each month)

We are ready to use Excel to calculate the payments necessary to save \$100,000.

	A	B	C	D	E	F	G
1			Excel Variable				
2	Annual Interest Rate:	0.07					
3	Monthly Interest Rate:	0.005833	rate				
4							
5	Number of months (periods)	216	nper				
6							
7	Present Value	0	pv				
8							
9	Future Value	100,000	fv				
10							
11	When are the deposits	0	type (0 for end of month or 1 for beginning of month)				
12							
13							
14	Payment		PMT				
15							

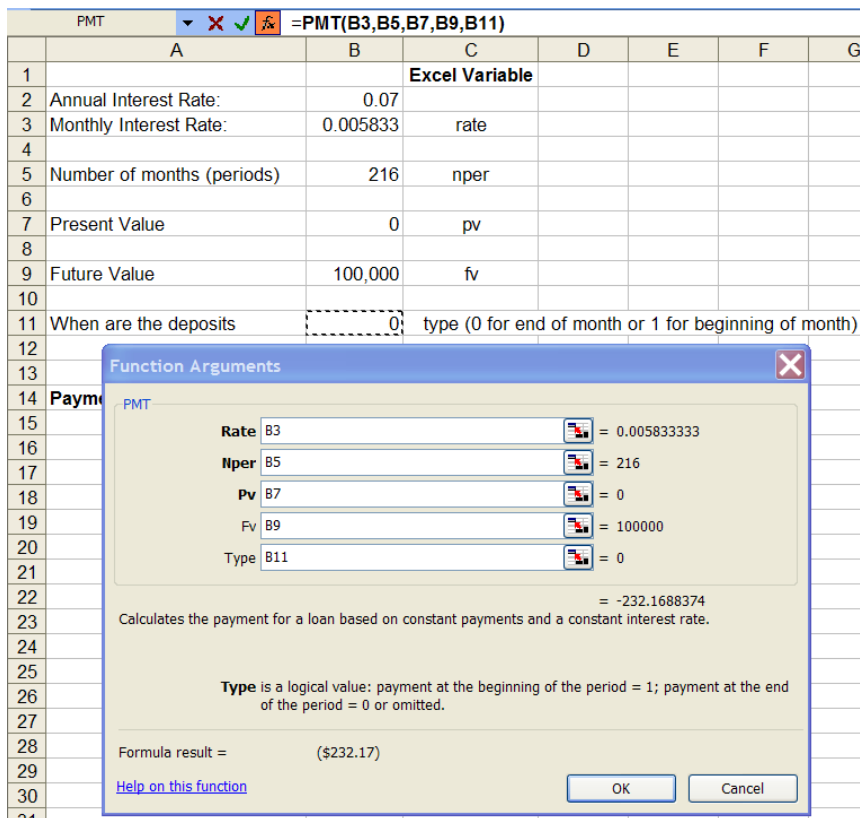
To keep things straight, type in each variable name and calculate or input the value of each variable. Although the annual interest is not used by the PMT formula, I have included it for completeness and to use in calculating the rate (= B2/12). Input the value of each variable, remember, use formulas when ever possible.

As before, click on the *fx* button shown here.



The Insert Function screen will appear. Select the PMT option shown by the blue line. Then click the OK button.

The following new screen will appear. Click on the box following the word “Rate.” Either type in the cell number for the monthly interest rate (B3) or click on cell B3. Continue with this process until all the boxes have the correct address. Then click the OK button.



The formula result is (\$232.17), which is consistent with our earlier calculations.

Why is the payment a negative number? We can tell it is negative because the parenthesis. Recall that Excel represents cash flows *in* as positive numbers and cash flows *out* as negative numbers. Although safely in the bank, Excel considers this deposit flowing out of your pocket and into the banks.

Solving Financial Problems (Continued)

Although saving money is preferred to borrowing money, there may be a time when borrowing money must be considered. This next section discusses installment loans and credit cards. Mortgages will be discussed in the next section.

Definition:

For any loan, the **principal** is the amount of money owed at any particular time. Interest is charged on the loan principal. To pay off a loan, you must gradually pay down the principal. Thus, in general, every payment should include all the interest you owe plus some amount that goes toward paying off the principal.

A loan that you pay off with equal regular payments is called an **installment loan**.

Installment Loans

Calculating Installment Loan Payments Using a Formula

Suppose you borrowed \$1,200 at an annual interest rate of APR = 12%, or 1% per month. What would your monthly payment be in order to pay back the money and interest in 6 months? The amount is not obvious, but we can calculate it with the **loan payment formula**.

$$PMT = \frac{P \left(\frac{APR}{n} \right)}{\left[1 - \left(1 + \frac{APR}{n} \right)^{(-nY)} \right]}$$

PMT = regular payment amount
P = starting loan principal (amount borrowed)
APR = annual percentage rate
n = number of payment periods per year
Y = loan term in years

In our example:

P = \$1,200

APR = 12%

n = 12 (because payments are monthly and there are 12 monthly periods per year)

Y = 0.5 year (the problem states 6 months but the variable Y must have units of years)

$$PMT = \frac{1,200 \left(\frac{0.12}{12} \right)}{\left[1 - \left(1 + \frac{0.12}{12} \right)^{(-12 \cdot 0.5)} \right]} = \frac{1,200(0.01)}{\left[1 - (1 + 0.01)^{-6} \right]} = \frac{12}{\left[1 - 0.942045235 \right]} = \$207.06$$

The monthly payments would be \$207.06.

When the first payment is made, some of the payment goes toward the principal and most goes to pay the interest. The loan principal is gradually paid down over the life of the loan. As the loan principal goes down the amount of interest which is charged each month also goes down allowing more to go toward the principal. In summary, the portions of installment loan payments going toward principal and toward interest vary as the loan is paid down. Early in the loan term, the portion going toward interest is relatively high and the portion going toward principal is relatively low. As the term proceeds, the portion going toward interest gradually decreases and the portion going toward principal gradually increases.

Using Excel (the Payment (PMT) Formula) to Calculate Installment Loan Payments

Already we have used the Excel Payment (PMT) Formula to calculate the deposits needed in a savings plan. Now the PMT formula can be used to calculate payments for an installment loan. Recall that the syntax for PMT is:

$$\text{PMT}(\text{rate}, \text{nper}, \text{pv}, [\text{fv}], [\text{type}])$$

Let us look at our problem again. You borrowed \$1,200 at an annual interest rate of APR = 12%. What would your monthly payment be in order to pay back the money and interest in 6 months? First, determine the values for each of the variables.

rate = 0.12 / 12 (remember Excel needs a monthly rate and not the annual interest rate)

nper = 6 (you hope to pay the loan back in 6 monthly payments)

pv = \$1,200 (the present value of the funds is \$1,200 which you presently have on hand)

fv = 0 (in the future you hope to owe nothing)

type = 0 (the payments will be made at the end of each month)

	A	B	C	D	E	F	G
1			Excel Variable				
2	Annual Interest Rate:	0.12					
3	Monthly Interest Rate:	0.01	rate				
4							
5	Number of months (periods)	6	nper				
6							
7	Present Value	1200	pv				
8							
9	Future Value	0	fv				
10							
11	When are the deposits	0	type (0 for end of month or 1 for beginning of month)				
12							
13							
14	Payment	(\$207.06)	PMT				
15							

Refer back to the last section for details of executing the PMT function in Excel. But using the values set forth here, the result of the Excel PMT function is \$207.06 which is the same as the hand calculated value. However, you will notice in Excel that the PMT is a negative number. Recall that this is because the monthly payments are flowing out of your pocket and Excel represents out going money with a negative.

Calculate Total Amount Paid and Total Interest Paid for Installment Loans

Now that we have calculated the payment, let us consider how much you will pay over the lifetime of the loan? And what is the total interest you will pay on the loan?

The total amount paid will be \$207.06 per month for 6 months.

Total Amount Paid = $\$207.06 \times 6 = \$1,242.36$

Of this total amount, \$1,200 pays off the principal. The rest, or \$42.36, represents interest paid.

Interest Paid = $\$1,242.36 - \$1,200.00 = \$42.36$

Choices of Rate and Term

You will usually have several choices of interest rates and loan terms when seeking a loan. For example, a bank might offer a 3-year car loan at 8%, a 4-year loan at 9%, and a 5-year loan at 10%. You will pay less total interest with the shortest-term, lowest rate loan, but this loan will have the highest monthly payments.

Thus, you will have to evaluate your choices and make the decision that is best for your personal situation. This table compares the car loan options assuming the car cost \$20,000.

	3-yr. 8% loan	4-yr. 9% loan	5-yr. 10% loan
PMT	\$626.73	\$497.70	\$424.94
Total Paid	\$22,562.18	\$23,889.64	\$25,496.45
Total Interest Paid	\$2,562.18	\$3,889.64	\$5,496.45

Usually shorter term loans have a lower interest rate than longer term loans. This is because there is less risk to loan for a shorter time. The risk of non payment causes the interest rate to go up. The following factors can affect the risk of the loan and the interest rate.

- Length of Loan (Shorter loans are less risky)
- Credit (A person with good credit looks less risky)
- Collateral (Home Loans are less risky, where Signature Loans are more risky)

Credit Cards

Credit card loans differ from installment loans in that you are not required to pay off your balance in any set period of time. Instead, you are required to make only a minimum monthly payment that generally covers all the interest but very little principal (if any). As a result, it takes a very long time (if ever) to pay off your credit card loan if you make only the minimum payments. If someone wishes to pay off their loan in a particular amount of time, they should use the loan payment formula to calculate the necessary payments.

Most credit cards have very high interest rates compared to other types of loans (no collateral, indefinite length of loan). If a person misses a payment, he will probably be charged a late fee that is added to his principal, thereby increasing the amount of interest due the next month.

Credit Card Example:

Adam has a credit card balance of \$2,300 with an annual interest rate of 21%. He decides to pay off his balance over 1 year. How much will he need to pay each month? What is the total paid in the year? What is the total interest paid in the year? Assume he will make no further credit card purchases.

$$P = \$2,300$$

$$APR = 21\%$$

$$n = 12 \text{ (because payments are monthly and there are 12 monthly periods per year)}$$

$$Y = 1 \text{ year}$$

$$PMT = \frac{P \left(\frac{APR}{n} \right)}{\left[1 - \left(1 + \frac{APR}{n} \right)^{(-nY)} \right]} = \frac{2,300 \left(\frac{0.21}{12} \right)}{\left[1 - \left(1 + \frac{0.21}{12} \right)^{(-12*1)} \right]} = \frac{2,300(0.0175)}{\left[1 - (1 + 0.0175)^{-12} \right]} = \frac{40.25}{\left[1 - 0.812058 \right]}$$

$$PMT = \$214.16$$

Adam must pay \$214.16 per month to pay off the balance in 1 year. The total amount paid will be \$2,569.92 (which is calculated by \$214.16 x 12). The total interest paid will be \$269.92 (which is calculated by (\$2,569.92 - \$2,300.00)).

Continuing the Credit Card Example:

Suppose Adam can get a personal loan at a bank at an annual interest rate of 10%. Should he take this loan and use it to pay off his \$2,300 credit card debt? Adam's goal is to pay as little as possible.

$$P = \$2,300$$

$$APR = 10\%$$

$$n = 12 \text{ (because payments are monthly and there are 12 monthly periods per year)}$$

$$Y = 1 \text{ year}$$

$$PMT = \frac{P \left(\frac{APR}{n} \right)}{\left[1 - \left(1 + \frac{APR}{n} \right)^{(-nY)} \right]} = \frac{2,300 \left(\frac{0.10}{12} \right)}{\left[1 - \left(1 + \frac{0.10}{12} \right)^{(-12*1)} \right]} = \frac{2,300(0.00833)}{\left[1 - (1 + 0.00833)^{-12} \right]} = \frac{19.16667}{\left[1 - 0.905212 \right]}$$

$$PMT = \$202.21$$

If Adam takes the bank loan, he must pay \$202.21 per month for 1 year to the bank. The total amount paid will be \$2,426.52 (which is calculated by \$202.21 x 12). The total interest paid will be \$126.52 (which is calculated by (\$2,426.52 - \$2,300.00)).

Let us compare the two situations: pay the credit card company for 1 yr or get a bank loan for 1 yr.

	Credit Card	Bank	Delta
PMT	\$ 214.16	\$ 202.21	\$ (11.95)
Total Paid	\$ 2,569.92	\$ 2,426.52	\$ (143.40)
Total Interest	\$ 269.92	\$ 126.52	\$ (143.40)

By going to the bank, Adam will pay \$11.95 less each month. And Adam will pay \$143.40 less interest in the year. Should Adam take the bank loan? Are there any drawbacks for taking the bank loan?

Sometimes banks charge fees for establishing the loan in addition to the interest being paid. Also some banks charge early payoff penalties. Make sure you consider all the fees.

Solving Financial Problems (Continued)

In this section, we will address mortgages.

Definitions:

The simplest type of home loan is a **fixed rate mortgage**, in which you are guaranteed that the interest rate will not change over the life of the loan. Most fixed rate loans have a term of either 15 or 30 years, with lower interest rates on the shorter-term loans.

There is also an **adjustable rate mortgage (ARM)**, in which the interest rate you pay changes whenever rates in the financial markets change. You will usually hear ARMs referred to as a 7 year ARM or a 5 year ARM. In this case the interest rate is fixed for the first 7 years or 5 years, respectively. After the set period of time the interest changes according to the finance markets. Most ARMs include a rate cap that cannot be exceeded. Why would someone be interested in an ARM? The interest rate is usually lower than the fixed rate mortgage.

Historically these are the primary types of mortgages and the only two discussed in your text. However, in the last 10 years new types of mortgages have emerged in an attempt to get more people to buy homes. Two will be discussed: the interest only loan and the option payment (or sub-prime) loan.

The **interest only mortgage** allows a person to only pay the interest portion of their loan for a period of time (like 5 years). After the set period of time, the loan usually becomes an adjustable rate mortgage subject to the going rate in the financial market. The benefit to this loan is a lower monthly payment. Disadvantages include a greater amount of interest is paid and no principle is paid.

The **option payment (or sub-prime) mortgage** allows a person to choose one of three amounts when paying their loan. The lowest option may be as low as 1% of the amount borrowed. Lenders offer this mortgage in order to get the monthly payments as low as possible. The disadvantage is that if the lowest option is the one being paid, then the amount owed actually goes up each month (an example will be shown later). As with the interest only mortgage, the option payment mortgage is limited to a set number of years and then it is adjusted.

Most lenders charge fees, or **closing costs**, at the time you take out a loan. Some fees are referred to as **points**. A point is 1% of the loan amount.

Fixed Rate Mortgages

Calculating Fixed Rate Mortgages Using a Formula

Suppose you borrowed \$100,000 to buy your new town house. The bank offers a 30-year fixed rate loan at an annual interest rate of $APR = 8\%$. What would your monthly payment be? Use the **loan payment formula**.

$$PMT = \frac{P \left(\frac{APR}{n} \right)}{\left[1 - \left(1 + \frac{APR}{n} \right)^{(-nY)} \right]}$$

PMT = regular payment amount
P = starting loan principal (amount borrowed)
APR = annual percentage rate
n = number of payment periods per year
Y = loan term in years

In our example:

P = \$100,000

APR = 8%

n = 12 (because payments are monthly and there are 12 monthly periods per year)

Y = 30 year

$$PMT = \frac{100,000 \left(\frac{0.08}{12} \right)}{\left[1 - \left(1 + \frac{0.08}{12} \right)^{(-12 \cdot 30)} \right]} = \frac{100,000(0.006667)}{\left[1 - (1 + 0.006667)^{-360} \right]} = \frac{666.67}{\left[1 - 0.091432472 \right]} = \$733.76$$

The monthly payments would be \$733.76 (this does not include escrow, property tax, PMI or insurance).

Over the 30-year life of the loan, the total payments are

$$30 \text{ yr} \left(\frac{12 \text{ mo}}{1 \text{ yr}} \right) \left(\frac{\$733.76}{1 \text{ mo}} \right) = \$264,150$$

The total payments for this \$100,000 loan would be about \$264,000 – more than 2 ½ times the starting principal.

Calculating Fixed Rate Mortgages Using Excel

In Excel use the Payment (PMT) Formula to calculate the monthly payment for the fixed rate mortgage. Recall that the syntax for PMT is:

$$\text{PMT}(\text{rate}, \text{nper}, \text{pv}, [\text{fv}], [\text{type}])$$

Using Excel (the Payment (PMT) Formula) to Calculate Installment Loan Payments

Already we have used the Excel Payment (PMT) Formula to calculate the deposits needed in a savings plan. Now the PMT formula can be used to calculate payments for an installment loan. Recall that the syntax for PMT is:

$$\text{PMT}(\text{rate}, \text{nper}, \text{pv}, [\text{fv}], [\text{type}])$$

Closing Costs

Comparing Closing Costs When Choosing a Mortgage

Great Bank offers a \$100,000, 30-year, 8% fixed rate loan with closing costs of \$500 plus 2 points. Big Bank offers a lower rate of 7.9%, but with closing cost of \$1,000 plus 2 points. Evaluate the two options.

What is the difference in closing costs?

What is the difference in monthly payments?

We just calculated a payment of \$733.76 for an 8% loan on \$100,000. Now let us do the same for the 7.9% loan.

P = \$100,000

APR = 7.9%

n = 12 (because payments are monthly and there are 12 monthly periods per year)

Y = 30 year

$$PMT = \frac{100,000 \left(\frac{0.079}{12} \right)}{\left[1 - \left(1 + \frac{0.079}{12} \right)^{(-12 \cdot 30)} \right]} = \frac{100,000(0.006583)}{\left[1 - (1 + 0.006583)^{-360} \right]} = \frac{658.33}{\left[1 - 0.094209747 \right]} = \$726.81$$

The monthly payments for the 7.9% loan would be \$726.81.

Comparing the two, we see that the 7.9% loan has a payment which is \$6.95 less per month (\$733.76 - \$726.81). The lower monthly payment is good, but is it worth the extra \$500 which you would have to pay today in order to get this lower rate.

Dividing \$500 by \$6.95 per month, we determine the time it will take to recoup the extra \$500:

$$\frac{\$500}{\$6.95 / mo} = 71.94mo \left(\frac{1yr}{12mo} \right) \approx 6yr$$

Thus, it will take you about 6 years to save the extra \$500 that Big Bank charges up front. The question then becomes, "how long will you have this loan?" If you think it's likely that you will sell or refinance within 6 years, you should not pay the higher closing costs. However, if you

expect to keep the loan for a long time, the extra closing costs might be worth it. If you keep the loan for the full 30 years (360 months), you'll save $360 \times \$6.95 = \$2,502$ in monthly payments over the life of the loan – far more than the extra \$500 you pay for the lower rate today. It is your option.

Comparing the Fixed Rate, ARM, Interest Only, and Option Payment Mortgages

Comparing the Fixed Rate and ARM Mortgages

These mortgages are difficult to compare because the ARM interest rate fluctuates. If you are considering a 7 year ARM, then the first 7 years can be compared using the *loan payment formula*. And for a 5 year ARM, the first 5 years can be compared. The ARM will have a lower interest rate. Why? Because - there is less risk for the lender. In choosing between a fixed rate and ARM, the question becomes: “How long will you have this loan?” If the time that you are going to have the loan will be shorter than or close to the 7 or 5 years (i.e. the stable part of the ARM), the ARM is a good choice. If the time will be much longer, then the ARM is probably not a good choice.

Comparing the Fixed Rate, Interest Only and Option Payment Mortgages

In this example, we will use Excel in class to consider a \$400,000 loan with an APR of 8%. Since the *interest only* and *option payment* mortgages are normally adjusted after 5 years, we will compare the results at the end of the first 5 years.

	Amount you still owe on the house	Monthly PMT	Interest Paid in 5 Years
30-Year Fixed	\$ 380,676.64	\$ 2,935.06	\$156,382.92
Interest Only	\$ 400,000.00	\$2,666.67	\$160,000.00
Option Pmt	\$ 567,992.71	\$333.33	\$20,000.00

The option payment results are based on an option to pay only 1% which is the most extreme case, however, it makes the point that the amount owed on the house increases each month. A rate “to good to be true” indeed may be. There may be new laws at this time; however, it has not always been required of the lenders to disclose that the amount owed was increasing.